The effect of wing surface area on the horizontal displacement of a paper aeroplane.

Physics.
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1 Introduction

When I was small I would always construct small aeroplanes with my friends. Some models worked better than others, and the most curious wing designs actually worked, whilst others surprisingly did not. Now that I am older and interested in studying aerospace engineering, I wanted to explore the impact wing surface area has on the aeronautical performance of an aeroplane. In order to keep everything as simple as possible, I decided to use a paper aeroplane, and build my own miniature catapult to include aspects of mechanical engineering.

1.1 Background Information

The flight of a paper aeroplane depends on four factors, lift, downforce, drag, thrust [3]. The thrust is provided by the catapult in this experiment, and drag occurs because air resistance exists. Downforce is simply the weight of the aeroplane, while lift is generated due to air moving under the wings while in flight [3]. Keeping everything else constant, the higher the surface area, the higher the lift will be [1]. This is because of the direct proportionality which can be expressed as:

$$L = c \cdot A$$

Where L is the lift generated, c is a constant, and A is the area of the wing [1]. In an optimum scenario, this increase in lift will be proportionally greater than the increase in drag [5]. The ratio between lift and drag is referred to as the L/D ratio [5]. This is equivalent to the glide ratio in gliding aircraft, the ratio of horizontal displacement to vertical displacement at constant speed [5]. As such, when the L/D decreases, the horizontal displacement will also increase.

1.2 Research Question

How does the horizontal displacement (in m) of the paper aeroplane depend on the surface area of its wings ($234,\ 230,\ 228,\ 226,\ 224,\ 222\ \text{cm}^2 \pm 1.00\ \text{cm}^2$) when it is launched from a catapult at a height of 179 ± 1cm?

1.3 Variables

1.3.1 Independent and Dependent Variables

As an independent variable, I was changing the surface area of the paper aeroplane, with the following values: 234, 230, 228, 226, 224 and 222 $\text{cm}^2 \pm 1.00\ \text{cm}^2$.

As the dependent variable, I measured the horizontal displacement of the paper aeroplane; how far it flew. This quantity is given in cm, and accurate to 3 cm.

1.3.2 Control Variables

The temperature of the room was kept constant (25 ± 1 °C). The windows and doors were closed whenever possible, such that no air of significantly different temperature could enter or exit. This is because an increase in air temperature decreases the air density reducing the lift [4], which ultimately grounds the aeroplane faster.

The catapult was kept constant, and the same rubber bands were used throughout the experiment. The experiment is an example of projectile motion, and the aim was to find how the horizontal component (displacement) changed when the vertical component was altered (lift due to surface area). This means that launching the aeroplane at different initial velocities or with different techniques also impacts the horizontal component, which is not fair testing.

The launch height was kept constant by placing the catapult at the same height in each trial. If the catapult height was variable, the horizontal displacement would vary, also resulting in unfair testing.

The mass of the aeroplane was kept constant (2.0 g ± 0.1 g). Because the launcher was always at the same height, the gravitational acceleration was kept constant at approximately 9.81ms^2 [6]. This means that a variation in mass will cause a different magnitude downward/graviational force, resulting in the net downward force being higher or lower.

The measuring equiptment was also kept constant by using the same tape for all trials. No tapes are identical and therefore give slightly deviating measurements.

1.4 Hypothesis

I hypothesise that the smaller the surface area of the wings, the smaller the displacement. This is because a smaller surface area implies less lift [1]. As the gravitational force remains constant, less lift results in a higher magnitude downwards net force, grounding the aeroplane faster. Because the displacement is measured at the point where the aeroplane first touches down, a quicker decline will decrease its horizontal displacement.

2 Materials and Methodology

2.1 Setup

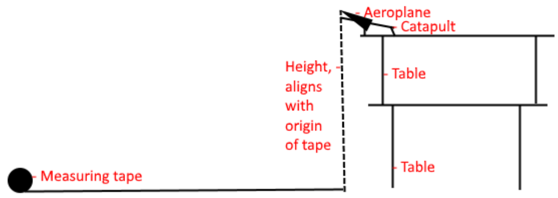

Figure 1: Diagram showing experiment setup

(a) Primed catapult on desk (b) Stacked desk setup

Figure 2: Pictures of setup used in experiment

Figure 1 shows the overall setup of the experiment. Important here is the height, which was 179 ± 1 cm. Notice that the catapult markings on the table are not visible in 2a due to the low quality quality of the picture. Observe the markings on the floor for the origin of the tape in figure 2b. Not in the picture is the heavy weight that fixed the tape origin to the right position.

2.2 Materials

- "Arrowhead" paper aeroplane, folded from blank, recycled A4 paper
- High platform (such as tables safely stacked on top of eachother)
- Paper aeroplane catapult, self built out of LEGO technic beams
- Profissimo pack of 100 rubber bands
  - 2x "green" rubber bands
  - 1x "yellow" rubber band
  - 1x "blue" rubber band
- Chalk
- Measuring tape, RSL Tools, 15 m, uncertainty unknown
- Ruler and set square, both Herlitz, (16 cm, 14 cm respectively), uncertainty unknown
- Balance, Kern Emb, ± 0.1 g, maximum 400 g.
- Heavy object to hold tape origin in place, in this case an alternator

2.3 Method

2.3.1 Reducing the Surface Area

In order to reduce the surface area by a certain amount (a), it was required of me to derive a formula to find the correct width at which I need to fold. This width is the only unknown variable, and must be solved for.

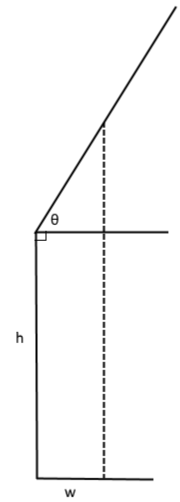

Figure 3: Diagram of the left wing of the aeroplane

In figure 3 the dotted line represents the line that needs to be folded to reduce the surface area by a. θ is the angle between the line normal to the height and the hypotenuse of the triangle. Therefore, in order to figure out what value w needs to be, solve for the positive solution of the following equation:

$$w^2 \tan(\theta) + 2wh - 2a = 0$$

Two key things are important here. Firstly, there are two wings on a paper aeroplane, so the value of a must be equal to half of the total desired surface area reduction. Secondly, it is recommended to solve this equation graphically using a GDC.

2.4 Execution of Experiment

1. Set up the experiment.
2. Prime the catapult and place the aeroplane inside.
3. Launch the catapult, and measure the horizontal displacement to the point of first touchdown. Write this down.
4. Repeat steps 2 to 3 five more times, such that 6 trials can be achieved.
5. Reduce surface area of the aeroplane by $4, 6, 8, 10, 12$ cm^2 and for each repeat steps 2 to 4.

2.5 Example Calculations

This is an example calculation for calculating how to reduce the surface area by 6.0 cm. Measuring the dimensions and angle of one of the wings yielded the following results: $a = 3, h = 4.2, \theta = 67°$

$$w \tan(67°) + (2 \cdot 4.2 \cdot w) - (2 \cdot 3) = 0 \rightarrow w_1 \approx -4.18,\ w_2 \approx 0.61$$

Rejecting w_1 as it is negative, the width would have to be ≈ 0.61 cm.

2.6 Environmental, Safety and Ethical Concerns

Environmental, safety and ethical concerns were incorporated into the planning of this experiment.

The catapult was built out of LEGO and can be re-used after the experiment. The paper from which the aeroplane was folded is recycled and can be recycled again, in order to prevent demand for further deforestation. Minimal paper was used to ensure a reduction of waste.

The catapult launched the aeroplane down an empty corridor, ensuring that nobody was in its flight path. The plane itself had minimal sharp edges, with only one point being noticeably sharp. This poing did not pose a danger as it faced the flight path, which was always clear during the experiment. It was handled in a manner such that no paper cuts were possible. The rubber bands of the catapult were tense, but not strong enough to significantly injure its operator in the unlikely case of a malfunction. The table was firmly held in place by a sturdy block of wood, such that it could not move, let alone fall down and hurt anybody.

No humans, animals or other living beings were hurt physically or emotionally by this experiment.

Surface Area (cm^2 $\pm$ 1 cm^2)	Horizontal Displacement ($\pm$ 3 cm)					
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
234	244	296	317	270	280	294
230	239	250	262	252	240	253
228	239	298	297	290	296	277
226	285	250	286	270	286	272
224	317	329	301	323	283	291
222	319	338	315	350	298	247

Table 1: Surface area of wings vs horizontal displacement of aeroplane

Surface Area (cm^2 $\pm$ 1 cm^2)	Avg. Horiz. Displ. ($\pm$ 3 cm)	St. Dev.
234	284	23*
230	249	8
228	283	20*
226	275	13*
224	307	17*
222	311	33*

Table 2: Surface area vs. average horizontal displacement
* An extra digit was added to account for a larger standard deviation

3 Results

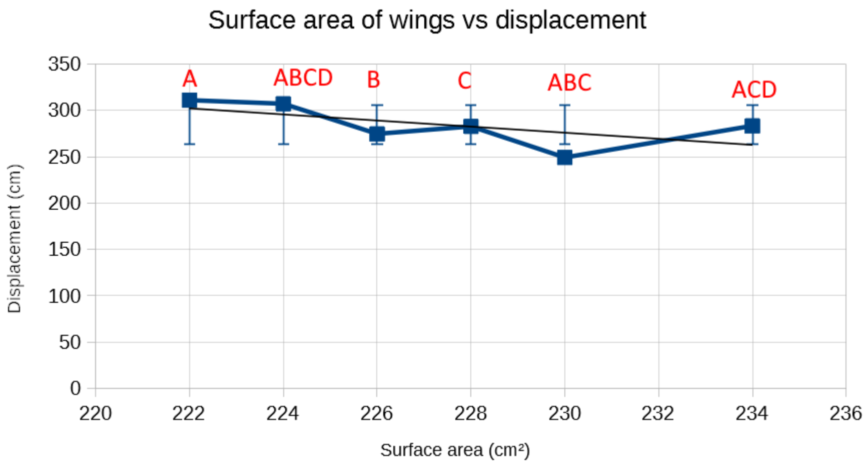

Figure 4: Surface area of wings vs horizontal displacement (enlarged)

3.1 Interpretation

The graph's trendline shows a negative relationship between the surface area and the displacement. As the surface area increases, the displacement decreases. There is a strong deviation from the trendline at 230 cm^2.

Additionally, the point at 234 cm^2 appears to be higher than expected. Unfortunately the difference between it and the rest of the data points is 4 cm^2, rather than the usual 2 cm^2. This makes it very hard to estimate what the data inbetween 230 and 234 cm^2 could be. However, due to at least one statistical difference between these, it could be that this point is also statistically different. The larger difference exists due to the nature of the paper aeroplane, 234 cm^2 is the original surface area, and a removal of 1 cm^2 per wing is not possible factoring in inaccuracies.

3.2 Statistics

Running a Student t-test on the data using mode 1 and type 2 yields results mapped to figure 5. Moreover, result groups are shown in red in figure 4. The headers in bold show the independent variables that were compared, the numbers below the p-value of the dependent variables. A p-value of ≤ 0.05 indicates there is no significant difference between the values.

222 and 224	**222 and 226**	**222 and 228**	**222 and 230**	**222 and 234**
0.005124997	0.481277752	0.238921061	0.045184077	0.07782547
224 and 226	**224 and 228**	**224 and 230**	**224 and 234**	
0.003634905	0.001845851	0.000019472	0.001149546	
226 and 228	**226 and 230**	**226 and 234**		
0.24132888	0.034203324	0.068426202		
228 and 230	**228 and 234**			
0.003270248	0.022760709			
230 and 234				
0.411201136				

Figure 5: Student's t-test on the data

This shows that whilst the points at 222 cm^2 and 234 cm^2 are not significantly different, there are significant differences between points lying in this rage. As an example, 226 cm^2 and 228 cm^2 are significantly different to each other, as indicated by the different groups.

4 Evaluation

4.1 Conclusion

As the surface area of the aeroplane's wings decreased, the displacement, for the majority, increased. This does not hold true for some areas as there was no statistically significant difference, due to the small range. This is presumably due to a decrease in drag for the areas in between, the aeroplane being more streamline in shape. This decrease in drag resulted in a greater net force in the direction of travel. Concurrently, the lower surface area decreased the lift, increasing the net downward force. Due to the increase in net thrust being larger than the increase in net downforce, the aeroplane travelled further horizontally. This occurred to the point where the plane displaced the most, but towards the end of the trials the lack of surface area caused a decrease in displacement. Referring to the background research, it appears that the L/D or glide ratio decreased, rather than the expected increase. This is unusual for gliders that have their wing surface area increased.

4.2 Evaluating the Hypothesis

The hypothesis does not hold true. I had hypothesised that as the surface area decreases, the displacement decreases. In the experiment, there was a significant increase, followed by a decrease insignificant to the initial value. Nonetheless, at an unknown point, the lack of surface area and thus lift will negatively impact the displacement. This was presumably just before the last value.

4.3 Literature Comparison

As aforementioned, assuming that the L/D ratio decreased, the experiment complies with the literature results: the plane's horizontal displacement will be decreased. Drag plays a role (as stated in 4.1), and experiments have found that alternating the winglets of a paper aeroplane influences its drag [3]. It is also mentioned that drag increases with the surface area [2], which corrolates with my data.

4.4 Strengths

The main strength was the constant mass throughout the experiment. Because the wings were folded rather than cut off, the mass remained constant. This ensured that the weight was constant, allowing the horizontal displacement to be tested fairly. Moreover, the plane itself was also kept constant, such that all aerodynamic properties were the same throughout all the trials.

The launching of the plane was a strength, as every launch executed the same. The catapult's location was marked on the table, so that in the case it got moved it could be put back into the correct position. The same rubber bands were used throughout the experiment, so the force delivered to push the plane did not differ. On top of that, the angle of attack (the angle between the plane and the horizontal) was also kept constant.

4.5 Limitations

Primarily the limitation is the small range of the independent variable. The range is too small to draw any definite conclusions and to characterize the behaviour of paper aeroplanes when the wing surface area is changed. The first and last values for the independent variable are not statistically significant, as such an extension of range would be required to see if there are any changes in behaviour beyond the current range.

Measuring the point of first impact precisely was difficult, and a small bit of estimation was involved ($\pm$ 3 cm). This of course increased the error margin significantly. To combat this, a sandpit can be used, where the point of first impact is clearly visible.

The aeroplane's path was also often not normal to the table where it was launched. This did not impact measurement, but the factor that made the aeroplane turn, such as air currents, is something that should be in control. This can be done by conducting the experiment in a room with tight seals so no outside draft could come in.

The rubber band may have loosened slightly during trials. This then impacts the initial velocity of when the plane starts gliding. To mitigate this, regular testing with a Newton meter can be done (i.e. at a force of x N the stretch should be y centimetres).

An unknown weakness is what caused the deviation from the trend line/random error for one of the points. Because the cause is unknown, it is difficult to pinpoint improvements. As a general thought, ensuring that the trials are conducted in the same manner and triple checking equiptment alignment may contribute towards elminating this.

4.6 Extension

Firstly, it can be experimented as to how the horizontal displacement changes when the angle of attack is changed. An example research question for this could be: How does the horizontal displacement (in cm) of a paper aeroplane depend on the angle of the aeroplane to the horizontal (0°, 15°, 30°, 45°, 60°, 75°)?

Secondly, the lift depends on the temperature of the air [4]. It could be experimented as to how that has an effect on the horizontal displacement. An example research question for this could be: How does the horizontal displacement (in cm) of a paper aeroplane depend on the temperature of the air (10°, 15°, 20°, 25°, 30°, 35°)?

Thirdly, it would be interesting to find the point where the increased streamline due to reduced surface area no longer compensates for the lack of lift. This would be the vertex of the surface area to displacement graph.

References

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[6] Ambler Thompson and Barry N. Taylor. *Guide for the Use of the International System of Units (SI). National Institute of Standards and Technology Special Publication 811 (2008 Edition).* National Institute of Standards and Technology / U.S. Department of Commerce, 2008.